Exam 3 Fall 2009 2048H

Name: SOLUTION

For score keeping purposes only

Problem 1

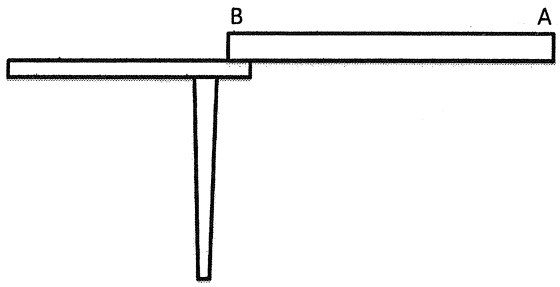
Problem 2

Problem 3

Problem 4

Problem 1 (25 points, 5 points each)

A uniform stick of mass M and length L is suspended horizontally with end B on the edge of the table and the other end A is held by a hand. Point A is released at t=0 suddenly. $I_{stick}=1/3\ ML^2$



(a) Before t=0, what is the force at A and B to maintain the static equilibrium?

Following problems asks the relevant values right after release of point A by hand.

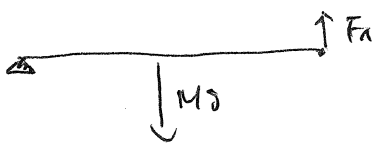
(b) What is the torque about B?

(c) What is the angular acceleration about B?

(d) What is the vertical acceleration of the center of the mass?

(e) Calculate the force at point B?

a)



$$F_A = \frac{Mg}{2}$$

b)

$$\vec{\tau} = -\frac{MgL}{2}$$

c)

$$\vec{\tau} = I\vec{\alpha} \qquad I = 1/3\,ML^2$$

$$-\frac{MgL}{2} = \frac{1}{3}ML^2\alpha$$

$$-\frac{3}{2}\frac{g}{L} = \alpha$$

d)

$$a = 1/2\,\alpha = 1/2\left(-\frac{3}{2}\frac{g}{L}\right)$$

$$= -\frac{3}{4}g$$

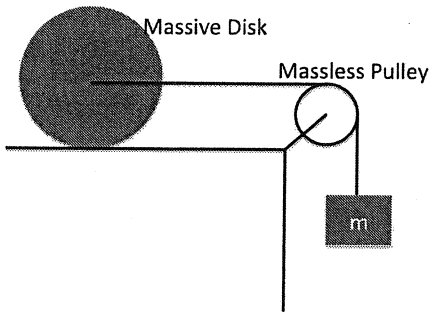
e)

$$m a_{CM} = \sum F =$$

$$-m\frac{3}{4}g = -mg + F_A$$

$$\boxed{F_A = \frac{1}{4}gM}$$

Problem 2 (25 points)



Massive disk of radius R and mass M is being pulled by a rope. The disk rolls without slipping. Rope is attached to a counterweight of mass m via a massless pulley of radius r. $I_{disk}=1/2\ MR^2$. The system starts at rest and starts moving because of the gravitational force on the counterweight.

(a) Is energy conserved? (3 points)
(b) Is angular momentum conserved? (3 points)

At some point mass m would have dropped by height h

(c) Find the velocity for the counter weight in terms of m,g,h,r,M,R, and I. (19 points)

a) YES

b) NO

c) $mgh = \frac{1}{2}Mv^2 + \frac{1}{2}Iw^2$

$v = wR \qquad w = \frac{v}{R}$

$mgh = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\frac{v^2}{R^2} + \frac{mv^2}{2}$

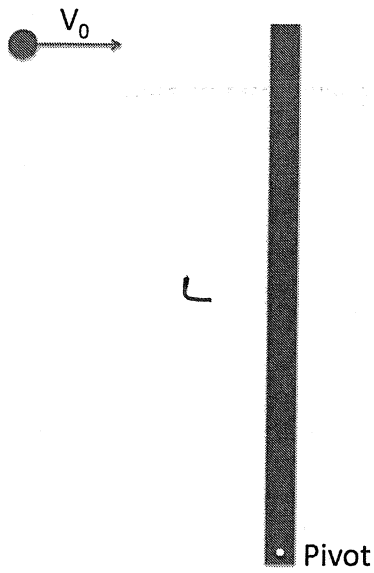
$mgh = \left(\frac{1}{2}Mv^2 + \frac{1}{4}Mv^2\right) + \frac{mv^2}{2}$

$mgh = \left(\frac{3}{4}Mv^2\right) + \frac{mv^2}{2}$

$v^2 = \frac{mgh}{\left(\frac{3}{4}M + \frac{m}{2}\right)}$

$v = \sqrt{\frac{mgh}{\frac{3}{4}M + \frac{m}{2}}}$

Problem 3 (25 points)



A ball with mass m collides with a rod of exactly same mass m and length L. The rod is pivoted as shown in the figure and the moment of inertia is $1/3\ mL^2$ about the pivot. Collision is elastic.

(a) What are the conserved quantities? (5 points)
(b) Calculate the final velocity of the ball in terms of V_0. (10 points)
(c) Calculate the final angular velocity of the rod. (10 points)

a) ANGULAR MOMENTUM AND ENERGY

b)

$$L_i = L_f$$

$$-mV_0L = -mV_FL + I\omega \Rightarrow mL(V_F - V_0) = I\omega$$

$$\frac{mL(V_F - V_0)}{I} = \omega$$

$$E_i = E_F$$

$$\frac{1}{2}mV_0^2 = \frac{1}{2}mV_F^2 + \frac{1}{2}I\omega^2$$

$$mV_0^2 - mV_F^2 = I\omega^2$$

$$mV_0^2 - mV_F^2 = I\,\frac{m^2L^2(V_F - V_0)^2}{I^2 I}$$

$$I = \tfrac{1}{3}mL^2$$

$$m(V_0^2 - V_F^2) = \frac{m^2L^2(V_F - V_0)^2}{\frac{1}{3}mL^2}$$

$$\tfrac{1}{2}m(V_0^2 - V_F^2) = \tfrac{3}{4}m(V_F - V_0)^2$$

$$V_0^2 - V_F^2 = 3(V_F^2 - 2V_0V_F + V_0^2)$$

$$0 = 4V_F^2 - 6V_0V_F + 2V_0^2$$

$$= V_0^2 - 3V_0V_F + 2V_F^2$$

$$V_F = \frac{3V_0 \pm \sqrt{9-8}}{4} V_0$$

$$= \cancel{V_0} \text{ OR } \boxed{\tfrac{1}{2}V_0}$$

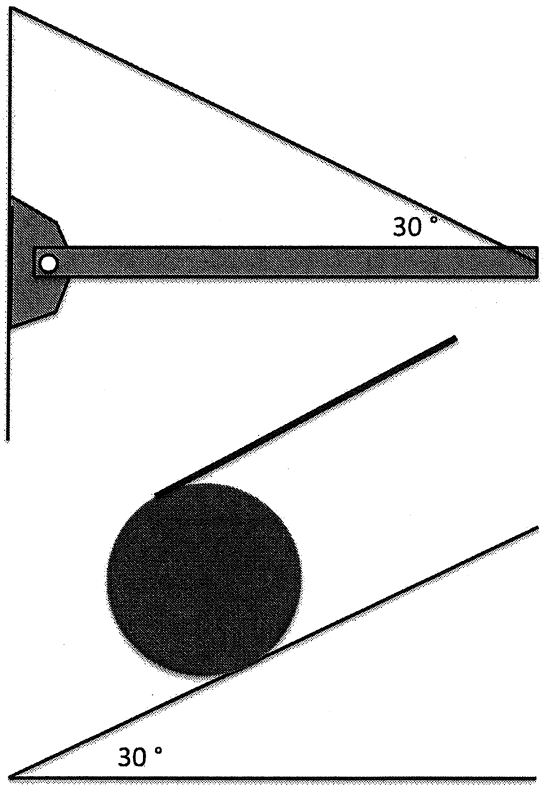
NOT SENSIBLE

c) $$\frac{mL(V_F - V_0)}{I} = \omega$$

$$+\frac{mL\,-\frac{1}{2}V_0}{\frac{1}{3}mL^2} = \omega$$

$$\frac{-\frac{1}{2}mLV_0}{\frac{1}{3}mL^2} = \omega \quad \boxed{\omega = -\frac{3}{2}\frac{V_0}{L}}$$

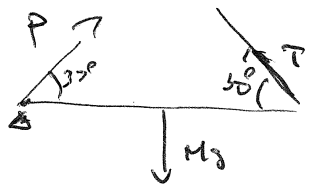
Problem 4 (25 points)



Massive plank of mass M and length L is being supported by a rope and a pivot point as shown in the figure. The plank is in static equibrium.
(a) Calculate tension of the rope. (6 points)
(b) Calculate Force applied by the pivot point (6 points)

Massive disk of mass M and radius R is supported by a rope on an incline surface with friction as shown in the figure.

(c) Calculate the normal force given that the surface static friction coefficient is μ. (6 points)
(d) Calculate the tension of the rope. (7 points)

a) 

$\tau = 0$

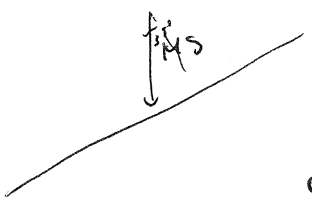
$\circlearrowleft \; T \sin 30^\circ \times L = \frac{MgL}{2}$

$T \; \frac{1}{2} \; R = \frac{Mg \, R}{2}$

$$\boxed{T = Mg}$$

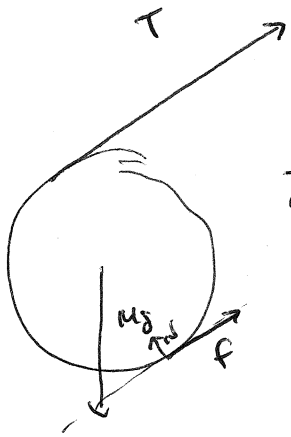
b) PIVOT POINT HAS TO BE SYMMETRIC IN THIS CASE

$P = Mg$ 30° UP AS INDICATED

c) 

$N = Mg \cos 30^\circ = Mg \frac{\sqrt{3}}{2}$

d) 



$$\sum \vec{\tau} = 0$$

$$T = f$$

$$f = \mu N$$

$$f = \boxed{\mu M g \frac{\sqrt{3}}{2} = T}$$